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7. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht

Abgabe: Mi. 11.01.2017 12:15 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Dreiergruppen erfolgen.

Aufgabe 13 (20 Punkte): *Feedback cooling of a trapped ion*

The goal of this exercise is to obtain a master equation that models the experiment in *Phys. Rev. Lett.* **96**, 043003 (2006), where the position of an ion in a trap was cooled by means of feedback action on the trap electrodes. As partially derived in exercise 9, the master equation for the one dimensional motion of a trapped ion in front of a mirror has the form

$$(1) \quad \dot{\rho}(t) = \mathcal{L}_0\rho(t) = -i[\nu b^\dagger b, \rho(t)] + \Gamma(N+1)\mathcal{D}[b]\rho(t) + \Gamma N\mathcal{D}[b^\dagger]\rho(t) + \frac{\gamma}{2}\mathcal{D}[1+\eta(b+b^\dagger)]\rho(t),$$

where ν is the trap frequency, N is the steady state occupation number, Γ the rate of the cooling, γ the pumping rate into the mirror mode and η the Lamb-Dicke parameter and $\mathcal{D}[b]\rho$ is a Lindblad term: $\mathcal{D}[b]\rho = b\rho b^\dagger - \frac{1}{2}\left(b^\dagger b\rho + \rho b^\dagger b\right)$. The special form of the last term is due to the fact that the mirror mode is not a mode in free space, on the one hand, and measured jumps on that mode imply a jump of the form $1+\eta(b+b^\dagger)$ on the vibrational degree of freedom.

(a) (Extra 10) With help of the stochastic Schrödinger equation for jump detections, derive the master equation for the evolution of the density matrix conditioned on the measurement of the position of the ion via the mirror mode

$$d\rho_c(t) = \mathcal{L}_0\rho_c(t)dt + \sqrt{2\gamma\eta^2}\mathcal{H}\rho_c(t)dW,$$

with the superoperator $\mathcal{H}\rho_c(t) = z\rho_c(t) + \rho_c(t)z - 2\langle z\rangle_c(t)\rho_c(t)$, the operator $z = b + b^\dagger$ and $\langle z\rangle_c(t) = Tr\{z\rho_c(t)\}$.

(b) (10) In order to affect the motion of the ion, the feedback term to be added to the master equation conditioned on the measurement is, in Stratonovich form,

$$(2) \quad -i\tilde{G}I(t)[z, \rho_c(t)],$$

with a feedback function $I(t) = \left(\gamma\eta\langle p\rangle_c(t)dt + \sqrt{\frac{\gamma}{2}}dW_\Xi\right)\cos(\nu t)$ and dW_Ξ a Wiener increment. Make use of the Stratonovich-Ito rule to incorporate the term to the conditioned master equation.

(c) (Extra 10) Find the unconditional form of the feedback master equation

$$(3) \quad \begin{aligned}\dot{\rho} = &-i[\nu b^\dagger b, \rho(t)] + \Gamma(N+1)\mathcal{D}[b]\rho(t) + \Gamma N\mathcal{D}[b^\dagger]\rho(t) \\ &- i\frac{\tilde{G}\gamma\eta}{4}[z, p\rho + \rho p] - \frac{\tilde{G}^2\gamma}{16}[z,[z,\rho]],\end{aligned}$$

where $p = i(b + b^\dagger)$ and the tilde refers to the interaction picture.

(d) (10) Derive the following expression for the final occupation of the vibrational degrees of freedom

$$(4) \quad \langle n\rangle_{ss} = \frac{N + \eta\gamma\tilde{G}(2N-1)/2\Gamma + \gamma\tilde{G}^2/8\Gamma}{1 + 2\eta\gamma\tilde{G}/\Gamma}.$$