Prof. Dr. Tobias Brandes
Dr. Javier Cerrillo

8. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht

Abgabe: Mi. 18.01.2017 12:15 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Dreiergruppen erfolgen.

Aufgabe 14 (20 Punkte): *Strong-Coupling Bosonic Transport*

Let us model energetic transport between two bosonic reservoirs strongly coupled through a two level system. For this, the polaron master equation provides a compact perturbative approach. The counting-field-resolved Hamiltonian describing the system is

$$\begin{aligned} H &\equiv H_S + H_B + H_{SB} \\ &= \frac{1}{2}\Delta\sigma_x + \sum_{q;\nu=L,R} \omega_{q\nu} a^\dagger_{q\nu} a_{q\nu} + \sigma_z \sum_{q;\nu=L,R} \gamma_{q\nu}(a_{q\nu} + a^\dagger_{q\nu}), \end{aligned}$$

where $\sigma_{x,z}$ correspond to the Pauli matrices and $a_{q\nu}$ is the annihilation operator associated to mode q of bath ν.

(a) (5) Transform the Hamiltonian into the polaron picture with help of the unitary

$$S = \exp\left[\sigma_z \sum_{q;\nu=L,R} \frac{\gamma_{q\nu}}{\omega_{q\nu}}(a_{q\nu} - a^\dagger_{q\nu})\right],$$

so that it takes the form

$$\begin{aligned} H' &\equiv H'_S + H'_B + H'_{SB} \\ &= +\frac{1}{2}\Delta\sigma_x + \sum_{q;\nu=L,R} \omega_{q\nu} a^\dagger_{q\nu} a_{q\nu} + (\sigma^+ D + H.c.), \end{aligned}$$

with $2\sigma^+ = \sigma_x + i\sigma_y$. Determine the form of D.

(b) (10) Introduce counting fields χ_ν associated to the energy of bath ν, $H'_\nu \equiv \sum_q \omega_{q\nu} a^\dagger_{q\nu} a_{q\nu}$. Confirm that the correlation function relevant for the derivation of the master equation has the form

$$(1) \qquad C(t,\chi_L,\chi_R) = \exp\left\{\sum_{q;\nu=L,R} \frac{\gamma^2_{q\nu}}{\omega^2_{q\nu}}\left[n_{q\nu} e^{i\omega(t+\chi_\nu)} + (n_{q\nu}+1)\, e^{-i\omega(t+\chi_\nu)}\right]\right\},$$

and determine the coefficients therein.

(c) (5) What relationship exists between the energy counting of the transformed baths and counting in the original picture?