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9. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht

Abgabe: Mi. 25.01.2017 12:15 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Dreiergruppen erfolgen.

Aufgabe 15 (20 Punkte): *Continued fraction formalism*

The Mori continued fraction formalism[1] facilitates the analytic computation of correlation functions by means of an iterative procedure in Laplace space. Let us consider a simplification thereof in the case of the laser-driven two-level atom. The density matrix of the system may be modeled by the master equation

$$\frac{d}{dt}\rho(t) \;=\; -i[\Delta\sigma_z + \Omega\sigma_x, \rho] - \tfrac{1}{2}\gamma\Big\{\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_- - 2\sigma_-\rho\sigma_+\Big\}. \tag{1}$$

where σ_i correspond to the Pauli matrices, Δ is the detuning of the laser, Ω the Rabi frequency and γ the spontaneous emission rate.

(a) (5) Show that a superoperator $\mathcal{L}[A,B]\rho \equiv A\rho B$ may be expressed as the matrix-vector multiplication

$$\left(A \otimes B^T\right)\vec{\rho},$$

where $\otimes$ indicates tensorial product and $\vec{\rho}$ is the vectorized form of ρ where columns from left to right have been stacked one under the other.

(b) (2) By using the previous result, find the matrix that represents the Lindbladian operator of the master equation. Separate the matrix into a jump part $\mathcal{J}$ and an effective Hamiltonian part $\mathcal{H}_{eff}$.

(c) (3) Show that, in Laplace space

$$\hat{\rho}(s) = \frac{\mathcal{R}_{eff}}{1 - \mathcal{R}_{eff}\mathcal{J}}\rho(0), \tag{2}$$

where $\mathcal{R}_{eff} \equiv \frac{1}{s - \mathcal{H}_{eff}}$ is the resolvent of the effective Hamiltonian superoperator $\mathcal{H}_{eff}$.

(d) (2) Express the differential equation

$$\frac{d}{dt}\rho_{eff}(t) = \mathcal{H}_{eff}\rho_{eff}(t) \tag{3}$$

as a system of differential equations for the four coefficients of the effective density matrix.

(e) (3) By using the previous results, provide a nested fraction representation of the occupation of the excited state in Laplace space.

(f) (5) Write an analytic expression for the long-time limit of the excited state occupation. Show that the resolvent of the effective Hamiltonian H_{eff} (operator) is sufficient to obtain the same result.

[1] Mori, H., *Prog. Theo. Phys.* **34**, 399 (1965).