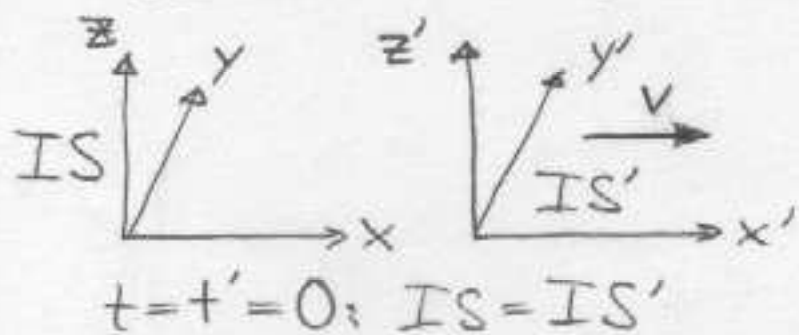


Lorentztrafo für "boost" in x-Richtung



allg. lineare Trafo:

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} \quad (16.15)$$

(i) Ursprung $x' = 0$ in IS: $x \stackrel{(16.15)}{=} -\frac{a_{21}}{a_{22}} ct \stackrel{!}{=} vt$

$$\rightarrow v = -\frac{a_{21}}{a_{22}} c \quad (1)$$

$$\stackrel{(16.15)}{\rightarrow} x' = a_{22} (x - vt) \quad (2)$$

(ii) Ursprung $x = 0$ in IS': $ct' \stackrel{(16.15)}{=} a_{11} ct$

$$x' = a_{21} ct$$

$$= \frac{a_{21}}{a_{11}} ct' \stackrel{!}{=} -vt'$$

$$\rightarrow v = -\frac{a_{21}}{a_{11}} c \quad (3)$$

$$(1) = (3) \rightarrow a_{11} = a_{22} \quad (4)$$

$$\stackrel{(16.15)}{\rightarrow} ct' = a_{22} \left(ct + \frac{a_{12}}{a_{22}} x \right) \quad (5)$$

(iii) (2) und (5) in

$$\begin{aligned} -(ct)^2 + x^2 &\stackrel{(16.9)}{=} -(ct')^2 + x'^2 = 1 \\ &= - \underbrace{\left(a_{22}^2 - a_{22}^2 \frac{v^2}{c^2} \right)}_{=1} (ct)^2 + \underbrace{\left(-a_{12}^2 + a_{22}^2 \right)}_{=1} x^2 \\ &\quad + \underbrace{\left(-a_{22} a_{12} - a_{22}^2 \frac{v}{c} \right)}_{=0} 2xct \end{aligned}$$

$$\rightarrow a_{22}^2 \left(1 - \frac{v^2}{c^2} \right) = 1$$

$$\rightarrow a_{22} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{in (2) \& (5)}$$

$$\rightarrow a_{12} = -a_{22} \frac{v}{c}$$

$$\rightarrow a_{12} = -\frac{v/c}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\rightarrow \boxed{\begin{aligned} x' &= \gamma (x - \beta ct) \\ ct' &= \gamma (ct - \beta x) \\ \beta &= \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} \end{aligned}}$$

(16.16)

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