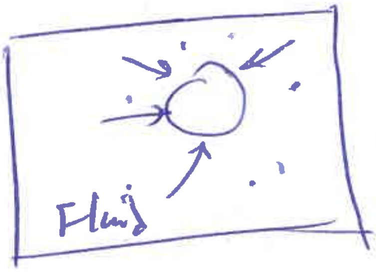
Active particles: Non equilibrium statistical physics. 25.11.2019

Equilibrium situation for an ordinary Brownian particles



Thermal fluctuations are responsible for the motion of the particle

$$m\ddot{x} = -\gamma \dot{x} + \sqrt{2k_BT}\,\gamma\,\xi(t) \quad , \langle \xi(t)\xi(s)\rangle = \delta(t-s)$$

overdamped: $m \to 0 \to \dot{x} = \sqrt{2D}\,\xi(t)$; $D = \frac{k_BT}{\gamma}$

→ Diffusion equation: $\frac{\partial}{\partial t} P(x,t) = D\frac{\partial^2}{\partial x^2} P(x,t)$

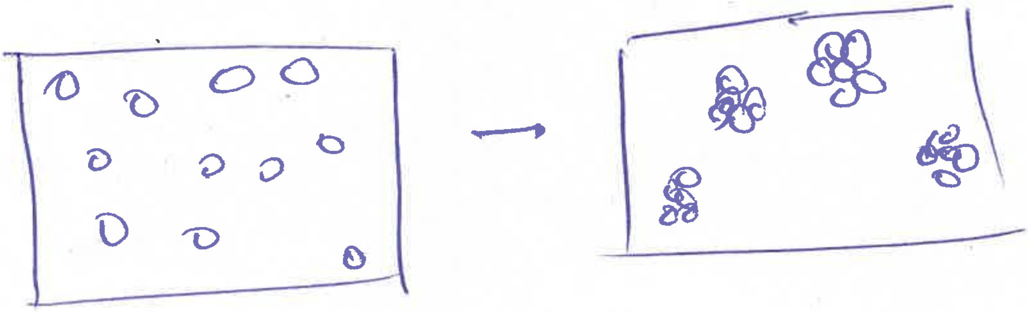
→ Active particles: Can convert local energy from their surrounding environment to perform directed motions;

* Directed motion drives the system far from equilibrium.

→ Artificial APs were synthesized to be able to study the situations under control.

Ex. Janus particle  → laser intensity → heats up pushes the particle!

→ Active particle show collective behaviors such as swarming, flocking, fish schools, ... → Theory for that may be found under motility induced phase separation (MIPS)



Models for active particles:

1 Active Brownian particle: → min here!

② Active Ornstein–Uhlenbeck particle: → $\dot{\underline{V}} = \gamma_R \underline{V} + \underline{\xi}(t)$

③ Run-Tumble particles: one-D, $\frac{dx}{dt} = \underline{V_0\, \sigma(t)}$

$$\eta(t) = \begin{cases} +V_0 \\ -V_0 \end{cases}$$

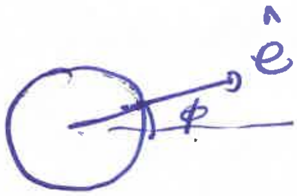
with some rate λ

$$\langle \eta(t_1)\eta(t_2)\rangle = V_0\, e^{-2\lambda|t_2-t_1|}$$

if $\lambda \to \infty$ → Gaussian noise:

$u \to \infty$

2-D Active Brownian particles:



$$\dot{\underline{r}} = V_0 \hat{e} + \underline{\xi}(t)$$

$$D_T = \frac{k_B T}{6\pi\eta R} \qquad D_R = \frac{k_B T}{8\pi\eta R^3}$$

$$\dot{x} = \sqrt{2D_T}\, \xi_T^1(t)$$
$$\dot{y} = \sqrt{2D_T}\, \xi_T^2(t)$$
$$\dot{\phi} = \sqrt{2D_R}\, \xi_R(t)$$

$$\langle x\rangle = \langle y\rangle = 0$$

coupling →

$$\dot{x} = V_0\cos\phi + \sqrt{2D_T}\, \xi_T^1(t)$$
$$\dot{y} = V_0\sin\phi + \sqrt{2D_T}\, \xi_T^2(t)$$
$$\dot{\phi} = \sqrt{2D_R}\, \xi_R(t)$$

$\langle x\rangle \neq 0$ initial and

$\phi(0) = 0$, $y(0) = 0$, $x(0) = 0$

$$\Rightarrow \langle x\rangle = \int_0^t \langle V_0 \cos\phi\rangle\, dt' + \int_0^t \langle \xi(t')\rangle\, dt' \sqrt{2D_T}$$

$$= \int V_0 \langle \cos\phi\rangle\, dt'.$$

$$\frac{\partial}{\partial t} p(\phi) = D_R \frac{\partial^2}{\partial\phi^2} p(\phi) \to p(\phi) = \frac{1}{\pi}\left(\frac{1}{2} - \sum \cos n\phi\, e^{-D_R t}\right)$$

$$\Rightarrow \langle \cos\phi\rangle = \int_0^{2\pi} p(\phi)\cos\phi\, d\phi \Rightarrow \langle x\rangle = \frac{V_0}{D_R}\left(1 - e^{-D_R t}\right)$$

$$\langle x\rangle_{t \gg \frac{1}{D_R} = \tau_R} = \underbrace{V_0 \tau_R \neq 0}, \quad \langle y\rangle = 0$$

→ length of directed motion before randomized!

$$\frac{dr}{dt} = v_0 \hat{e} + \xi(t)$$

$$\frac{d\phi}{dt} = \xi_R$$

$$p(r,\phi,t) = \langle \delta(r - r(t))\, \delta(\phi - \phi(t)) \rangle$$

$$\frac{\partial}{\partial t} p(r,\phi,t) = -v_0 \hat{e} \cdot \nabla p(r,\phi,t) - \frac{\partial}{\partial\phi} \langle \xi_R\, \delta(x - x(t)) \times \delta(\phi - \phi(t)) \rangle - \nabla \cdot \langle \xi_T\, \delta(x - x(t))\, \delta(\phi - \phi(t)) \rangle$$

Novikov's theorem

$$\Rightarrow \frac{\partial p}{\partial t}(r,\phi,t) = -v_0 \hat{e} \nabla p(r,\phi,t) + \frac{\partial^2}{\partial\phi^2} p + \Delta p(r,\phi,t)!$$