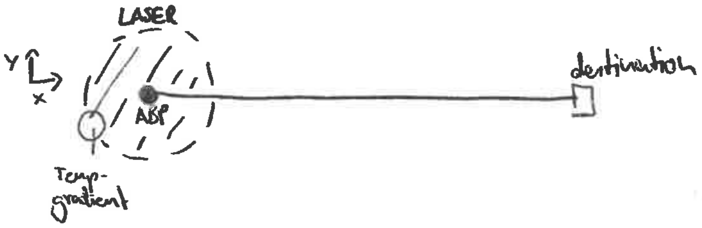
Theo 6 27.11.

Delayed feedback control of an Active Brownian particle



moves directed when shone on

1. Keep laser still in y-direction
2. Do not know $x(t)$, only $x(t-\delta)$ because of delay

Do not know Orientation of ABP either

→ if $(x(t) - x(t-\delta)) > 0$ → right direction → higher laser intensity

< 0 → wrong direction → lower laser intensity

$$V(x,y) = \frac{1}{2}\eta\left((x-x_0)^2+(y-y_0)^2\right)$$

Assumption: $V \propto I$

$$I = I_0\left(1+\beta(x(t)-x(t-\delta))\right)$$

$$v(t) = v_0\left(1+\beta(x(t)-x(t-\delta))\right)$$

$$\eta(t) = \eta_0\left(1+\beta(x(t)-x(t-\delta))\right)$$

→ SDE:

$$\dot{x}(t) = v_0\left(1+\beta(x(t)-x(t-\delta))\right)\cos\phi - \eta_0\left(1+\beta(x(t)-x(t-\delta))\right)\left(x(t)-x(t-\delta)\right) + \sqrt{2D_T}\,\xi_T^1(t)$$

$$\dot{y}(t) = v_0\left(1+\beta(x(t)-x(t-\delta)\right)\sin\phi - \eta_0\left(1+\beta(x(t)-x(t-\delta))\right)\left(y(t)\right) + \sqrt{2D_T}\,\xi_T^2(t)$$

$$\dot{\phi}(t) = \sqrt{2D_R}\,\xi_R(t)$$

with large delay: $\delta \gg \tau_R = \frac{1}{D_R}$ → Non-Markovian, control mech. fails

small delay: $\delta \ll \tau_R$ → $\beta(x(t)-x(t-\delta)) = \beta\,\delta\,\dot{x}(t) + O(\delta^2)\beta$ (ignore δ^2)

↓

$$\dot{x} = v_0\cos\phi + \beta\delta v_0^2\cos^2\phi - \eta_0 v_0\cos\phi + (1+\beta\delta v_0\cos\phi - \eta_0\delta)\sqrt{2D_T}\,\xi(t)$$

$$\dot{y} = v_0\sin\phi + \beta\delta v_0^2\sin\phi\cos\phi - \eta_0 y(1+\beta$$

$$\dot{\phi} = \sqrt{2D_R}\,\xi_R(t)$$

decouple DGLs and remove ∂^2

$$\dot{x} = V_0 \cos\phi + \delta\beta V_0^2 \cos^2(\phi) + \sqrt{2D_T}\, \xi_T$$

$$\dot{y} = V_0 \sin\phi + \beta\delta V_0^2 \cos\phi \sin\phi - \eta y_0 + \sqrt{2D_T}\, \xi_T$$

$$\dot{\phi}(t) = \sqrt{\frac{2}{\tau_R}}\, \xi_R$$

$\downarrow$ solve by coarsgraining ϕ

small $\begin{cases} \tau_R = k\epsilon \\ \delta = c\epsilon \end{cases}$ with $\frac{c}{k} \to 0$ bec. $\delta \ll \tau_R$

$$\langle r \rangle = V_0 \tau_R \quad \to V_0 \overset{!}{=} \frac{U}{\sqrt{\tau_R}} = \frac{U}{\sqrt{k\epsilon}}$$

$$\dot{x} = \frac{U}{\sqrt{k\epsilon}} \cos\phi + \beta \frac{c}{k} \cos^2\phi + \sqrt{2D_T}\, \xi_T^1(t)$$

$$\dot{y} = \frac{U}{\sqrt{k\epsilon}} \sin\phi + \beta \frac{c}{k} U^2 \cos\phi \sin\phi - \eta y + \sqrt{2D_T}\, \xi_T^2$$

$$\dot{\phi} = \sqrt{\frac{2}{k\epsilon}}\, \xi_R$$

Backward FP (?)/Backward Kolmogorov

$\hat{e} = \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$

$$\partial_t p = \frac{1}{k\epsilon} \partial_\phi^2 p + \left(\frac{U}{\sqrt{k\epsilon}} + \beta \frac{c}{k} U^2 \cos\phi\right) \hat{e} \nabla p - \eta_0 y \partial_y p + D_T \nabla p)$$

$p = p(r, \phi, t)$

Perturbation theorie:

$$\tilde{p} = p_0(r,t) + \sqrt{\epsilon}\, p_1(r,\phi,t) + \epsilon\, p_2(r,\phi,t) + \epsilon^{\frac{3}{2}} \ldots$$

seperate Operators: $L_0 = \frac{1}{k} \partial_\phi^2$

$L_1 = \frac{U}{\sqrt{k}} \hat{e}_1 \nabla$

$L_2 = \beta \frac{c}{k} U^2 \cos\phi\, \hat{e} \cdot \nabla - \eta_0 y \partial_y + D_T \Delta$

$\downarrow$

ϵ^{-1}: $L_0 p_0 = \frac{1}{k} \partial_\phi^2 p_0 = 0 \quad \to p_0(r,t) = \cancel{A\phi} + \beta(r,t)$ bestimmt nicht

$\epsilon^{\frac{1}{2}}$: $L_1 + L_0 p_1 = 0 = \frac{U}{\sqrt{k}} \hat{e} \cdot \nabla p_0 + \frac{1}{k} \partial_\phi^2 p_1 \quad \to p_1 = \sqrt{k}\, U \hat{e} \nabla p_0$

$\sqrt{\epsilon}$ ϵ^0: $L_1 p_1 + L_2 p_0 + L_0 p_2 = \frac{U}{\sqrt{k}} \hat{e} \nabla p_1 + \beta \frac{c}{k} U^2 \cos\phi\, \hat{e} \nabla p - (\eta_0 \partial_y + D_t \Delta) p_0 + \frac{1}{k} \partial_\phi^2 p_2 \quad = \partial_t p_0$

$$u(r,\phi,t) = \partial_t \varphi_0 - L_1 \phi_1 - L_2 \varphi_0 \qquad L_0 \varphi_2 = \frac{1}{\cdot} \partial_\phi^2 \varphi_2$$

$$\int_0^{2\pi} d\phi\, u(r,\phi,t)\, p = 0 \quad \Rightarrow \partial_t \varphi_0(r,\phi,t) = \left(\beta\delta \frac{V_0^2}{2} \partial_x - \varphi_0\, y\, \partial_y + \left(D_T + \tau_R \frac{V^2}{2}\right) D\right) \varphi_0$$

↓ massive Diffusion (hot passive particle)

$$\dot{x} = \beta\delta \frac{V^2}{2} + \sqrt{2D_T + V_0^2 \tau_R}\; \xi(t)$$

$$\dot{y} = -\eta\, y_0 + \sqrt{2D_T + V_0^2 \tau_R}\; \xi(t)$$

$$\Rightarrow \langle x \rangle = \beta\delta \frac{V^2}{2} t \quad \rightarrow \quad t = \frac{L}{\beta\delta \frac{V^2}{2}}$$ wenn L Strecke zum Ziel