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Projekte zur TP VI Vertiefung: Nichtgleichgewichtsstatistik

Durchführung

Die Projekte beinhalten Aufgaben aus verschiedenen Bereichen der Nichtgleichgewichtsstatistik und können nach eigenen Vorstellungen bearbeitet werden (Numerik, Analytik, Zusammenfassung der Literatur, Experimente ...). Die in jeder Projektbeschreibung aufgeführten Punkte können als Leitfaden dienen, Sie können aber auch in Absprache mit den BetreuerInnen eigene Ideen verfolgen.
Die Projekte sind so konzipiert, dass die Bearbeitung mit der angegebenen Literatur und dem Wissen aus der Vorlesung möglich ist. Bei einigen Projekten werden allerdings besondere Vorkenntnisse benötigt (z.B. MATLAB).
Zur vollständigen Bearbeitung gehören folgende Punkte:

1. Bearbeitung des Projekts in Zweier- oder Dreiergruppen
2. Präsentation der Ergebnisse in einem 15 minütigen Kurzvortrag (+5 Minuten Diskussion) am 10.02.2020 und 12.02.2020 . Wichtig ist hierbei in erster Linie die verständliche Darstellung. Beschränken Sie sich deshalb auf die zum Verständnis wesentlichen Punkte.
3. Abgabe einer schriftlichen Ausarbeitung mit vollständiger Dokumentation der Lösungswege und vollständigen Quellenangaben bis zum 21.02.2020. Auch hier steht die Verständlichkeit und übersichtliche Darstellung im Vordergrund. Der Umfang der Ausarbeitung soll fünf bis zehn Seiten umfassen.

Während der gesamten Bearbeitungszeit stehen Ihnen die BetreuerInnen des jeweiligen Projektes für Fragen zur Verfügung. Bitte machen Sie individuell Termine mit den Betreuern aus.

Projekt 1: *Role of noise for coherence-incoherence patterns in networks*

Betreuer: Anna Zakharova

One of the challenging issues concerning coherence-incoherence patterns is their formation in the presence of random fluctuations, which are unavoidable in real-world systems. The robustness of chimera states with respect to external noise has been studied only very recently [1]. An even more intriguing question is related to the constructive role of noise for partial synchronization patterns [2]. The goal of the present project is to provide a review on the role of random fluctuations for coherence-incoherence patterns.

- Perform a literature review about this topic based on the given references and the material cited therein. Understand what are chimera states and, in particular, amplitude chimeras, chimera death, and coherence-resonance chimeras.
- Compare FitzHugh-Nagumo system in the excitable regime and Stuart-Landau model. What are the differences and similarities?
- For which patterns do the random fluctuations play a destructive role?
- For which patterns do the fluctuations play a constructive role?

Literature

[1] Loos, S., Claussen, J. C., Schöll, E. and Zakharova, A., Chimera patterns under the impact of noise, Phys. Rev. E 93, 012209 (2016).
[2] Semenova, N., Zakharova, A., Anishchenko, V. S. and Schöll, E., Coherence-resonance chimeras in a network of excitable elements, Phys. Rev. Lett. 117, 014102 (2016).

Projekt 2: *Active Brownian particle vs active Ornstein-Uhlenbeck particle*

Betreuer: Mohsen Khadem

The capability of converting energy from surrounding environment into a directed motion derives actives particles far from equilibrium. This directed motion destroys the time reversibility at the microscopic length-scales and allows the active particles (AP) to exhibit behaviors which are impossible in thermal equilibrium systems. We will consider two different models of APs being i) active Brownian particle and ii) active Ornstein-Uhlenbeck particle, and compare the characteristic quantities calculated from these models. Comparing these quantities we will be able to check when these models are equivalent and how different parameters in both models are connected. Within this project, we will learn, how the standard techniques in stochastic processes could be utilized to explain complicated observations in the nature and laboratory. The following steps apply for both models:

- Write down the Langevin equation of motion for your model.
- Calculate the mean position of the particle using direct integration of the stochastic differential equation of the motion.

- Calculate the mean squared displacement (MSD) of the particle using direct integration of the stochastic differential equations. Plot the MSD.
- Derive the Fokker-Plank equation for the probability distribution function (PDF) of the particle displacement corresponding to the stochastic differential equation of the motion.
- Solve the Fokker-plank equations (applying the needed approximations). Plot the PDF at different time scales.
- Repeat steps two and three using the PDF in calculations instead of direct integration of the equation of the motion.

Literature

[1] Review article: Bechinger, C., Di Leonardo, R., Lwen, H., Reichhardt, C., Volpe, G., and Volpe, G., Active particles in complex and crowded environments. Reviews of Modern Physics, 88(4), 045006, (2016).
[2] Review article: Romanczuk, P., Br, M., Ebeling, W., Lindner, B., and Schimansky- Geier, L., Active Brownian Particles-From Individual to Collective Stochastic Dynamics . The European Physical Journal Special Topics, 202, (2012).
[3] Standard books or lecture notes for stochastic processes. Suggestion: https://courses.physics.ucsd.edu/2015/Fall/physics210b/LECTURES/CH02.pdf
[4] Sevilla, F. J., and Sandoval, M. Smoluchowski diffusion equation for active Brownian swimmers. Physical Review E, 91(5), 052150,(2015).

Projekt 3: *Stochastic resonance; Simulation of active Brownian particle in a double-well potential*

Betreuer: Mohsen Khadem

Stochastic resonance (SR) refers to a phenomenon whereby a weak signal can be amplified and optimized by the assistance of noise, see Ref. [1] for review. Three basic ingredients for such an effect to occur are: (i) an energetic activation barrier or, more generally, a form of threshold; in our case the double well potential; (ii) a weak coherent input (such as a periodic signal); (iii) a source of noise corresponding to inherit property of the system. A detailed theoretical description of SR for the case of the passive particle in a double-well potential in the presence of a periodic signal has been provided in the lecture and may also be found in Ref [2]. The aim of this project is to examine possible variations in SR when, instead of a passive particle, an active Brownian particle (ABP) is considered. In the ABP model [3], the particle experiences a self-propelling force in addition to the translational diffusion. This force results in a persistent motion of the particle whose direction changes by the rotational diffusion. Since both rotational and translational diffusions stem from the same origin being the thermal fluctuations, any variation in the noise would also affect the directed motion of the particles.

- Write down the Langevin equation of motion for your model. The potential is considered to be one-dimensional and affects only the x-direction.

- Calculate the stationary distribution of the particle position in x-direction, at two limits of small and large rotational diffusion coefficient (See Ref [4] for hint). Compare your results with that for a passive particle.
- Apply a weak periodic signal tilting the potential (as in the lecture).
- Run a set of simulations and compute the signal to noise ratio (SNR) as a function of noise. (The rotational and translational noises are proportional to each other)
- Investigate the effect of activity (v_0) on the SNR by tuning the vale of the activity.

[1] Gammaitoni L, Hänggi P, Jung P, Marchesoni F. Stochastic resonance. Reviews of modern physics. 1998 Jan 1;70(1):223, (1998).
[2] McNamara and Wiesenfeld, Theory of stochastic resonance, Physical Review A. 39, 4854 (1989).
[3] Sevilla, F. J., and Sandoval, M, Smoluchowski diffusion equation for active Brownian swimmers. Physical Review E, 91(5), 052150, (2015).
[4] Pototsky, A., Stark, Active Brownian particles in two-dimensional traps. Europhysics Letters, 98(5), 50004, (2012) .

Projekt 4: *Fluctuation-dissipation relation in non-equilibrium, a review*

Betreuer: Mohsen Khadem

Fluctuation-dissipation (FD) theorem allows one to express the linear response of a system to an external perturbation in terms of the spontaneous correlations, and its derivation is based on the detailed balance condition [1]. Developement of the FD relation within the context of equilibrium statistical mechanics of Hamiltonian systems has led to a misleading claim that its validation is only limited to systems in equilibrium [2]. The goal of this project is to study the generalized FD relations which hold also in non-equilibrium.

- Perform a literature search about this topic.
- Re-derive the generalized FD relation in [2].
- Examine the validity of the generalized FD relation above for the anomalous diffusion by relating the first and the second moments of the particle displacement.
- Examine the validity of the generalized FD relation above for the the case of an active particle modeled by energy depot [3].

[1] Sarracino, A., and A. Vulpiani. On the fluctuation-dissipation relation in non-equilibrium and non-Hamiltonian systems. Chaos: An Interdisciplinary Journal of Nonlinear Science 29, no. 8, 083132, (2019).
[2] Kubo's famous text: Fluctuation-dissipation theorem, R. KUBO, Department of Physics, University of Tokyo, Japan, http://www-f1.ijs.si/ ramsak/km1/kubo.pdf
[3] F. Schweitzer, W. Ebeling, and B. Tilch,Complex motion of Brownian particleswith energy depots,Phys. Rev. Lett. 80 , 5044–5047 (1998).

Projekt 5: *Coherence resonance in heterogeneous networks*

Betreuer: Leonhard Schülen

The counter-intuitive effect of coherence resonance describes a non-monotonic behavior of the regularity of noise-induced oscillations in the excitable regime, leading to an optimum response in terms of regularity of the excited oscillations for an intermediate noise strength [1]. Coherence resonance has been investigated in complex networks with local, nonlocal and global topologies as well as in random and small-world networks [2]. The phenomenon of coherence resonance has been also recently studied in a two-layer multiplex network [3]. In [2] and [3] the network elements are all assumed to be identical. In real-world systems, however, the elements are often non-identical. The goal of this project is to investigate the impact of heterogeneity on coherence resonance.

- Perform a literature search about this topic. Has coherence resonance been previously investigated in networks with heterogeneous elements?
- Reproduce the results (without delays) obtained in [2]. In particular, reproduce Fig. 2.
- Consider the case of inhomogeneous FitzHugh-Nagumo neurons in excitable regime. For example, a_i is normally distributed around the value 1.1. Make sure that that $a_i > 1 \; \forall i$ as this is the bifurcation point. Perform a simulation for $\sigma = 0.1$ and $D = 0.001$. What are the effects of the inhomogeneities?
- Try to get a similar curve as in Fig. 2, now for the inhomogeneities. Is the picture similar or does it deviate significantly? If so, is the effect dependent on the noise intensity/the coupling strength?

[1] A. Pikovsky and J. Kurths: Coherence resonance in a noise-driven excitable system, Phys. Rev. Lett. 78, 775 (1997).
[2] Masoliver, M., Malik, N. , Schöll, E. and Zakharova, A. , Coherence resonance in a network of FitzHugh-Nagumo systems: interplay of noise, time-delay and topology, Chaos 27, 101102, (2017).
[3] N. Semenova and A. Zakharova: Weak multiplexing induces coherence resonance, Chaos 28, 051104, (2018).

Projekt 6: *Coherence resonance in a globally coupled network*

Betreuer: Leonhard Schülen

The counter-intuitive effect of coherence resonance describes a non-monotonic behavior of the regularity of noise-induced oscillations in the excitable regime, leading to an optimum response in terms of regularity of the excited oscillations for an intermediate noise strength [1]. Coherence resonance has been investigated in complex networks with local, nonlocal and global topologies as well as in random and small-world networks [2]. In globally coupled networks, external noise on the units can cause a spike in individual oscillators. If enough oscillators cross the threshold, they pull the rest of the network with them, resulting in a spike of all neurons. This can be called an *avalanche*. The goal of this project is to determine the number of oscillators that need to cross the threshold in order to result in such an avalanche. The following steps are to be taken:

- Perform a literature search on coherence resonance in globally coupled networks. Except for Ref. 2, has coherence resonance been previously investigated in globally coupled networks?
- Reproduce the results (without delays) obtained in [2]. In particular, reproduce Fig. 5 e).
- Perform a simulation with $D = 0.0001$, i.e. outside of the coherence resonance regime. Create a space-time plot and/or a video for $t = 500$ time units and check for a spike of all neurons. How many neurons spiked right before that, i.e. caused the avalanche?
- Perform more simulations for different noise values (around $D = 0.0001$) and try to find an efficient method to count the spikes that cause the avalanche. Is the number fixed? Perform more simulations, but now with $D = 0.0001$ and varying the coupling strengths. What is the impact?
- Increase the number of oscillators. How many neurons need to spike now? Is the fraction A/N constant? (A is the number of neurons that trigger the avalanche and N the number of neurons.)

[1] A. Pikovsky and J. Kurths: Coherence resonance in a noise-driven excitable system, Phys. Rev. Lett. 78, 775 (1997).
[2] Masoliver, M., Malik, N. , Schöll, E. and Zakharova, A. , Coherence resonance in a network of FitzHugh-Nagumo systems: interplay of noise, time-delay and topology, Chaos 27, 101102 (2017).