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6. Übungsblatt zur Theor. Physik VI: Nichtgleichgewichtsstatistik

Abgabe: Mi. 18.12.2019. Die Abgabe erfolgt in **2er oder 3er Gruppen**.

Aufgabe 9 (20 Punkte): *Coherence Resonance*

Consider a generalized Van der Pol oscillator, extended by a quartic term in the nonlinear friction. If we additionally involve time-delayed feedback, it is described by the following equation:

$$\frac{d^2x}{dt^2} - \left[\varepsilon + \mu x^2 - x^4\right]\frac{dx}{dt} + \omega_0^2 x = K(x(t-\tau) - x(t)), \tag{1}$$

where x is the dimensionless variable, t is the dimensionless time, $\varepsilon \in \mathbb{R}$ and $\mu > 0$ are the parameters responsible for excitation and dissipation, respectively, ω_0 is the eigenfrequency of linear oscillations at the Hopf bifurcation, K is the strength of time-delayed feedback, τ is the delay time. Because of the quartic nonlinearity the system can exhibit simultaneously two limit cycles: a stable and an unstable one. The regime of coexistence of these two periodic orbits is limited from one side by a saddle-node bifurcation of limit cycles ($\varepsilon = -\mu^2/8$), and from the other side by a subcritical Hopf bifurcation ($\varepsilon = 0$). Besides two limit cycles, the regime $-\mu^2/8 < \varepsilon < 0$ contains a stable focus in the origin. For $\varepsilon < -\mu^2/8$ the only attractor is the stable focus. It becomes unstable for $\varepsilon > 0$ in a subcritical Hopf bifurcation at $\varepsilon = 0$.

(a) Averaging method for the deterministic dynamics

1) Find averaged equation for the evolution of the complex amplitude using the following ansatz:

$$x(t) = Re\{A(t)\exp(i\omega_0 t)\} = \frac{1}{2}\{A\exp(i\omega_0 t) + c.c.\}, \tag{2}$$

where $A(t)$ is a complex amplitude, and $c.c.$ denotes the complex conjugate $A^*\exp(-i\omega_0 t)$. Exploit the following assumptions: the amplitude of the oscillations is changing slowly on the time-scale of the period of oscillation, therefore, the averaging method (quasiharmonic reduction) can be applied to the Van der Pol equation; the delay τ is small, so that we can approximately set $A(t-\tau) \approx A(t)$ on the slow time scale of $A(t)$.

2) Write the equations for the amplitude and the phase dynamics separately. To do so transform to polar coordinates:

$$A = \rho\exp(i\phi), \tag{3}$$

where $\rho \geq 0$ is the amplitude and $\phi \in \mathbb{R}$ is the phase of oscillations. Find the non-trivial solutions of the amplitude equation.

3) From the amplitude equation for ρ define the new rescaled bifurcation parameter ε in dependence on τ. How does delay shift the bifurcation parameter? Interpret the results.

(b) Coherence resonance in generalized van der Pol equation with Gaussian white noise.
The generalized van der Pol equation can be re-written as a 2-variable dynamical system:

$$\begin{aligned} \frac{dx}{dt} &= y, \\ \frac{dy}{dt} &= [\varepsilon + \mu x^2 - x^4]y - \omega_0^2 x + \sqrt{2D}\xi(t) + K(x(t-\tau) - x(t)), \end{aligned} \tag{4}$$

where $\xi(t)$ is normalized Gaussian white noise: $\langle\xi(t)\xi(t+\tau)\rangle = \delta(\tau)$, $\langle\xi(t)\rangle = 0$, and D is the noise intensity.

6. Übung TPVI WS19/20

Calculate analytically (using the potential solution) the stationary probability distribution of the amplitude $a(t) = \sqrt{x^2(t) + y^2(t)}$. How does the distribution change for different values of τ? Calculate the correlation time $t_{cor} = \frac{1}{\Psi(0)} \int_0^\infty |\Psi(s)| ds$, where $\Psi(s)$ is autocorrelation function. How does the time delay influence coherence resonance?
Use the following parameter values: $K = 0.024$, $D = 0.003$, $\mu = 0.5$, $\varepsilon = -0.06$, $\omega_0 = 1$. Since the period of oscillations in the deterministic system without delay is $T = 2\pi$, take $\tau = 0.25T, 0.5T, 1T, 0.75T$.

Vorlesung:	• Montag 12:15 Uhr – 13:45 Uhr im EW 203. • Mittwoch 10:15 Uhr – 11:45 Uhr im EW 203.
Übung:	• Mittwoch, 14:00 – 16:00 Uhr im EW 229.
Anmeldung:	Die Punkteverteilung und Scheinvergabe zu der Vorlesung "Statistische Physik im Nichtgleichgewicht" erfolgt über das Moseskontosystem: `https://moseskonto.tu-berlin.de/moseskonto`.
Webseiten:	• Details zur Vorlesung, Vorlesungsmitschrift und aktuelle Informationen sowie Sprechzeiten auf der Webseite unter: `https://www.itp.tu-berlin.de/menue/lehre/lv/ws_201920/wahlpflichtveranstaltungen_master/statistische_physik_im_nichtgleichgewicht//`
Scheinkriterien:	• Mindestens 50% der Übungspunkte. (Abgabe in Dreiergruppen). • Bearbeitung und Vorstellung eines Projektes (Projektvorstellung in der letzten Vorlesungswoche). • Regelmäßige und aktive Teilnahme in der Übung.
Kontakte:	• Prof. Dr. Anna Zakharova, ER 244, 314-28948, `anna.zakharova@tu-berlin.de`, Sprechzeiten nach Vereinbarung • Dr. Mohsen Khadem EW 266, 314-28849, `jebreiilkhadem@physik.tu-berlin.de`, Sprechzeiten Do. 16:00-17:00