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7. Übungsblatt – TPVI: Theorie des Quantentransportes

Abgabe: Do. 05.12.2019 16:00 Uhr im Tutorium
Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen oder Dreiergruppen erfolgen.

Aufgabe 15 (40 Punkte): *Open two-level system*
Points (a), (c), (d) and (e) should be done analytically, for the others you may use any programming language (Mathematica, Python, etc..) but explain your answers.

The master equation of an arbitrary system coupled to ONE reservoir is

$$\dot{\rho} = \pi \sum_{\omega=\{\epsilon_r - \epsilon_{r'}\}} \gamma(\omega) \left(e^{-\beta\omega} \mathcal{D}\left[S_\omega\right]\rho + \mathcal{D}\left[S_\omega^\dagger\right]\rho \right),$$

where $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}\left\{A^\dagger A, \rho\right\}$ and $S_\omega = \langle r|S|r'\rangle |r\rangle\langle r'|$ and the states $|r\rangle$ are the eigenstates of the Hamiltonian of the system $H_S|r\rangle = \epsilon_r|r\rangle$.

Consider the following open two level system described by Hamiltonian

$$H = \omega_0\sigma_z + \Delta\sigma_x. \tag{1}$$

(a) (2 Punkte) What are the eigenenergies ϵ_r and eigenstates $|r\rangle$ of H?

(b) (2 Punkte) Plot the eigenenergies as a function of Δ.

The two-level system is coupled to two bosonic reservoirs $\nu \in \{L, R\}$ with Hamiltonian $H_B^{(\nu)} = \sum_k \omega_{k,\nu} b_{k,\nu}^\dagger b_{k,\nu}$. Their interaction is given by

$$H_I = \sigma_x \sum_{k,\nu} g_{k,\nu}(b_{k,\nu}^\dagger + b_{k,\nu}). \tag{2}$$

From now on assume $\Delta = 0$.

(c) (2 Punkte) Find H_I in the interaction picture.

(d) (2 Punkte) What are the possible frequencies $\omega = \{\epsilon_r - \epsilon_{r'}\}$ and operators S_ω for the two-level system?

(e) (4 Punkte) Taking $\gamma_\nu(\omega) = \frac{\Gamma_\nu(1+N_\nu)}{2\pi}$, write the master equation of the open two-level system.

(f) (4 Punkte) In superoperator form, the master equation can be written as $\dot{\vec{\rho}} = \mathcal{L}\vec{\rho}$, where $\vec{\rho}$ is the vectorization of the density matrix. Write $\mathcal{L}$ for the two-level system in matrix form.

(g) (8 Punkte) Considering N_ν now to be $N_\nu(\omega) = \frac{1}{e^{\beta_\nu\omega}-1}$ and parameters $\Gamma_R = 0.2\omega_0$, $\Gamma_L = 0.2\omega_0$ and $\beta_R\omega_0 = 4$ plot the steady state populations of the two-level system as a function of $\beta_L\omega_0$ between $\beta_L\omega_0 = 0$ and $\beta_L\omega_0 = 10$.

(h) (8 Punkte) Plot the stationary heat flow corresponding to the right reservoir ($\nu = R$) for the same parameters.

(i) (8 Punkte) Now consider the case $\omega_0 = 0$ and $\Delta \neq 0$ in Eq. (1). Plot the stationary heat flow corresponding to the right reservoir?

BONUS: (12 extra Punkte) Do points (c), (d), (e) for $\Delta \neq 0$.

Vorlesung:	• Do. 10:00 Uhr – 12:00 Uhr im EW 203. • Fr. 10:00 Uhr – 12:00 Uhr im EW 203.
Übung:	• Do. 16:00 Uhr –18:00 Uhr im EW 733.
Scheinkriterien:	• Mindestens 60% der Übungspunkte. • Regelmäßige und aktive Teilnahme am Tutorium.