Technische Universität Berlin – Institut für Theoretische Physik 4. Dezember 2019

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8. Übungsblatt – TPVI: Theorie des Quantentransportes

Abgabe: Do. 12.12.2019 16:00 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen oder Dreiergruppen erfolgen.

Aufgabe 16 (30 Punkte): *Trace distance and information back-flow*

The trace distance between two states is defined as

$$D(\rho_1, \rho_2) = \frac{1}{2}\|\rho_1 - \rho_2\|_1,$$

where $\|A\|_1 = \text{Tr}\sqrt{A^\dagger A}$.

(a) (3 Punkte) Show that for an arbitrary two level system we have

$$D(\rho_1(t), \rho_2(t)) = \sqrt{a_t^2 + |b_t|^2},$$

where $a_t = \rho_{++}^{(1)} - \rho_{++}^{(2)}$ and $b_t = \rho_{+-}^{(1)} - \rho_{+-}^{(2)}$.

(b) (2 Punkte) Calculate the trace distance between the states $\rho_1 = \frac{3}{4}|+\rangle\langle+| + \frac{1}{4}|-\rangle\langle-|$ and $\rho_2 = \frac{2}{3}|\phi_1\rangle\langle\phi_1| + \frac{1}{3}|\phi_2\rangle\langle\phi_2|$, with $|\phi_1\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ and $|\phi_2\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$

The master equation for the pure dephasing model is

$$\dot{\rho}(t) = \gamma(t)[\sigma_z\rho(t)\sigma_z - \rho(t)],$$

with $\gamma(t)$ a time-dependent dephasing rate.

(c) (2 Punkte) Solve the master equation for an arbitrary initial state.

(d) (3 Punkte) Show that the information flux $\sigma(t) = \frac{d}{dt}D(\rho_1(t), \rho_2(t))$ is given then by

$$\sigma(t) = -\frac{2\gamma(t)|b_0|^2 e^{-2\Gamma(t)}}{\sqrt{a_0^2 + |b_0|^2 e^{-2\Gamma(t)}}},$$

where $\Gamma(t) = 2\int_0^t \gamma(t')dt'$.

The expressions for dephasing rate and the spectral density are

$$\gamma(t) = \int_0^\infty d\omega\, J(\omega)\coth(\beta\omega/2)\frac{\sin(\omega t)}{\omega}, \qquad J(\omega) = \frac{\omega^s}{\omega_c^{s-1}}e^{-\omega/\omega_c},$$

where ω_c is a cutoff frequency and s is the Ohmicity parameter of the bosonic environment. For $s < 1$ the environment is called sub-Ohmic and for $s > 1$ super-Ohmic. The Ohmic case $s = 1$, is typically associated with Markovianity due to the non-structured form of $J(\omega)$. From now on consider the bath to be at zero temperature, $\omega_c = 1.0$ and use the programming language (Mathematica, Python, etc..) you prefer.

(e) (5 Punkte) Plot the dephasing rate in the time interval $t \in (0, 1)$ for $s = \{\frac{1}{2}, 1, 2, 3, 4, 5\}$. Can you infer something regarding the information flux?

(f) (5 Punkte) Plot the trace distance between the initial states $\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ and $\frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$ in the time interval $t \in (0, 4)$ for the same values of s.

(g) (5 Punkte) Plot the positive values of the information flux for the same parameters as (f).

(h) (5 Punkte) For which $s = \{\frac{1}{2}, 1, 2, 3, 4, 5\}$ is there a back flow of information? Should those spectral densities be associated to a Markovian or non-Markovian environment?

Vorlesung:	• Do. 10:00 Uhr – 12:00 Uhr im EW 203. • Fr. 10:00 Uhr – 12:00 Uhr im EW 203.
Übung:	• Do. 16:00 Uhr –18:00 Uhr im EW 733.
Scheinkriterien:	• Mindestens 60% der Übungspunkte. • Regelmäßige und aktive Teilnahme am Tutorium.