Technische Universität Berlin – Institut für Theoretische Physik 9. Dezember 2019

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# 8. Übungsblatt – TPVI: Theorie des Quantentransportes

**Abgabe: Do. 19.12.2019 16:00 Uhr im Tutorium**

*Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen oder Dreiergruppen erfolgen.*

**Aufgabe 17 (9 Punkte):** *Counting fields*

The moment generating function associated with the probability $p(\Delta n = n_t - n_0)$ of projectively measuring $N_\mathsf{B}$ at time $t$ obtaining $n_t$ and at time $0$ obtaining $n_0$ is

$$M(\chi) = \sum_{\Delta n} e^{i\chi\Delta n} p(\Delta n) = \mathsf{Tr}\left\{\rho(\chi, t)\right\}, \tag{1}$$

where $\rho(\chi, t)$ is the generalized density matrix and $\chi$ is a counting field. The reduced density matrix $\rho(t)$ of our system is recovered by setting $\chi$ to zero. The cumulant generating function of the particle current is then defined as

$$C(\chi) = \frac{d}{dt} \ln M(\chi), \tag{2}$$

so that the cumulants are obtained by simple differentiation

$$\frac{d}{dt}\left\langle\left\langle \Delta n^m \right\rangle\right\rangle = \left.\frac{\partial^m}{\partial(i\chi)^m} C(\chi)\right|_{\chi=0}. \tag{3}$$

Assume the generalized density matrix obeys an equation of the form $\partial_t \rho(\chi, t) = \mathcal{L}(\chi, t)\rho(\chi, t)$ where $\mathcal{L}(\chi, t)$ is a time-dependent superoperator and $\mathcal{L}(0, t)$ is trace preserving.

(a) (3 Punkte) Show that the current is given by

$$I(t) \equiv \frac{d}{dt}\left\langle\left\langle \Delta n \right\rangle\right\rangle = \mathsf{Tr}\left\{\mathcal{L}'(t)\rho(t)\right\},$$

where $\mathcal{L}'(t) = \frac{\partial}{\partial(i\chi)}\mathcal{L}(\chi, t)|_{\chi=0}$.

(b) (3 Punkte) Show that the noise is given by

$$S(t) \equiv \frac{d}{dt}\left\langle\left\langle \Delta n^2 \right\rangle\right\rangle = \mathsf{Tr}\left\{\mathcal{L}''(t)\rho(t) + 2\mathcal{L}'(t)\sigma(t)\right\},$$

where $\mathcal{L}''(t) = \frac{\partial^2}{\partial(i\chi)^2}\mathcal{L}(\chi, t)|_{\chi=0}$ and we have defined the traceless operator

$$\sigma(t) \equiv \frac{\partial}{\partial(i\chi)}\left[\rho(\chi, t)/\mathsf{Tr}\left\{\rho(\chi, t)\right\}\right]|_{\chi=0}. \tag{4}$$

(c) (3 Punkte) Taking the time derivative of equation (4) show that $\sigma(t)$ can be obtained from the auxiliary equation

$$\frac{d}{dt}\sigma(t) = \mathcal{L}'(t)\rho(t) - I(t)\rho(t) + \mathcal{L}(t)\sigma(t), \tag{5}$$

What is the initial condition for this equation?

**Aufgabe 18 (21 Punkte):** *Electron pumping*
Electron pumping consists in taking advantage of explicit time dependencies in the Hamiltonian of a system in order to transfer electrons between two different reservoirs kept at equal inverse temperature $\beta$ and chemical potential $\mu$. A simple model for an electron pump can be achieved by a simple SET with time-dependent energy and tunneling rates.

$$\frac{d}{dt}\begin{pmatrix} P_0(t) \\ P_1(t) \end{pmatrix} = \begin{pmatrix} -\Gamma_L(t)f_L(t) - \Gamma_R(t)f_R(t) & \Gamma_L(t)\left(1 - f_L(t)\right) + \Gamma_R(t)\left(1 - f_R(t)\right) \\ \Gamma_L(t)f_L(t) + \Gamma_R(t)f_R(t) & -\Gamma_L(t)\left(1 - f_L(t)\right) - \Gamma_R(t)\left(1 - f_R(t)\right) \end{pmatrix}\begin{pmatrix} P_0(t) \\ P_1(t) \end{pmatrix},$$

with $f(t) = f(\epsilon(t))$ a time-dependent Fermi distribution, $P_0(0,t) = 1 - n(t)$, $P_1(0,t) = n(t)$ and $n(t)$ is the occupation of the dot.

(a) (3 Punkte) By introducing a counting field associated with the left reservoir show that the current is given by

$$I(t) = \Gamma_{\mathsf{L}}(t)\left[f(t) - n(t)\right].$$

(b) (3 Punkte) Show that the noise is given by

$$S(t) = \Gamma_{\mathsf{L}}(t)\left[f(t) + n(t) - 2f(t)n(t) + 2f(t)\sigma_0(t) - 2(1 - f(t))\sigma_1(t)\right]$$

where $\sigma_0(t)$ and $\sigma_1(t)$ are the entries of operator $\sigma(t)$ defined in (4).

(c) (3 Punkte) Show that they follow the equations

$$\begin{aligned} \dot{\sigma}_0(t) &= -f(t)\Gamma_+(t)\sigma_0(t) + \left[1 - f(t)\right]\Gamma_+(t)\sigma_1(t) - I(t)(1 - n(t)) - \left[1 - f(t)\right]\Gamma_{\mathsf{L}}(t)n(t), \\ \dot{\sigma}_1(t) &= f(t)\Gamma_+(t)\sigma_0(t) - \left[1 - f(t)\right]\Gamma_+(t)\sigma_1(t) + \Gamma_{\mathsf{L}}(t)f(t)\left[1 - n(t)\right] - I(t)n(t), \end{aligned}$$

where $\Gamma_+(t) = \Gamma_R(t) + \Gamma_L(t)$.

The time dependence of the energy and tunneling rates is given by

$$\epsilon(t) = 20\Gamma\cos(\omega t), \quad \Gamma_L(t) = \frac{\Gamma}{2}\exp\left\{6\cos(\omega t - \phi) - 1\right\}, \quad \Gamma_R(t) = \frac{\Gamma}{2}\exp\left\{6\cos(\omega t) - 1\right\}.$$

Note that the functions are periodic with period $T = 2\pi/\omega$. Consider now the following parameters $\Gamma = 1$, $\beta = 100$, $\mu = 0$ and $\omega = 5 * 10^{-5}$.

(d) (4 Punkte) Plot the occupation of the dot for $\phi = \{0, \pi/2, -\pi/2\}$ between the times $t = 5T$ and $7T$.

(e) (4 Punkte) Plot the current for the same parameters.

The total pumped charge per period $Q$ and its fluctuations $\Delta Q^2$ can be obtained by integrating $I(t)$ and $S(t)$ over a period.

(f) (4 Punkte) Calculate $Q$ and $\Delta Q^2$ by integrating $I(t)$ and $S(t)$ from $t = 5T$ to $6T$.

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| Vorlesung: | • Do. 10:00 Uhr – 12:00 Uhr im EW 203. <br> • Fr. 10:00 Uhr – 12:00 Uhr im EW 203. |
| Übung: | • Do. 16:00 Uhr –18:00 Uhr im EW 733. |
| Scheinkriterien: | • Mindestens 60% der Übungspunkte. <br> • Regelmäßige und aktive Teilnahme am Tutorium. |