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10. Übungsblatt – TPVI: Theorie des Quantentransportes

Abgabe: Do. 09.01.2020 16:00 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen oder Dreiergruppen erfolgen.

Aufgabe 19 (30 Punkte): *Full counting statistics*

Consider the case of a quantum dot connected to two different electronic reservoirs and introduce a counting field in order to count positive when electrons enter the left reservoir.

(a) (2 Punkte) How does the probability for n electrons entering the left reservoir up to time $t = 0$ look?

(b) (5 Punkte) Calculate the probability for n electrons entering the left reservoir at steady state for the parameters $\mu_L = V/2,\ \mu_R = -V/2,\ V = 4\epsilon,\ \Gamma_L = 1.5\gamma,\ \Gamma_R = 2\gamma,\ \beta_L\epsilon = 0.5,\ \beta_R\epsilon = 1.5$ and $\gamma t = \{0.25, 2, 6, 15, 25\}$. Explain the observed behavior.

(c) (2 Punkte) Qualitatively explain how the probability distribution will look for the previous parameters if $V \to -V$.

Consider now that the single electron transistor is being monitored by a quantum point contact. The total dissipator for the SET is then

$$\mathcal{L}(\xi, \chi) = \mathcal{L}_{\mathsf{SET}}(\xi) + \mathcal{L}_{\mathsf{QPC}}(\chi), \tag{1}$$

where ξ is a counting field with respect to the left electronic reservoir, χ a counting field for the QPC, $\mathcal{L}_{\mathsf{SET}}(0)$ the dissipator of an SET and

$$\mathcal{L}_{\mathsf{QPC}}(\chi) = \begin{pmatrix} 1 & 0 \\ 0 & |1-\alpha|^2 \end{pmatrix} T \left[\frac{\tilde{V}}{e^{\beta\tilde{V}} - 1} \left(e^{-i\chi} - 1\right) + \frac{\tilde{V}}{1 - e^{-\beta\tilde{V}}} \left(e^{i\chi} - 1\right) \right], \tag{2}$$

where T is the transmission and parameters β and $\tilde{V}$ indicate temperature and bias voltage of the low transparency QPC.

(d) (2 Punkte) Show that the stationary state of the SET is unaffected by the QPC.

(e) (4 Punkte) Calculate the steady state current through the QPC.

(f) (5 Punkte) Show that at the limit of high QPC bias voltage ($\tilde{V} \to \infty$) the probability for n jump events if the SET is empty is given by

$$P_n(t) = \frac{e^{-\tilde{V}Tt}\,(\tilde{V}Tt)^n}{n!} \tag{3}$$

(g) (5 Punkte) Plot the actual probability distribution if the SET is empty, valid for all bias voltage, and compare with equation (3) for the following parameters $T = 1,\ t = 1,\ \beta = 0.2$ and $\tilde{V} = \{2, 8, 18\}$. How does the agreement between the two distributions depend on the temperature?

The cumulant generating function obeys the symmetry

$$C(\xi) = C(-\xi + i\alpha). \tag{4}$$

(h) (5 Punkte) Show that for the SET this symmetry can be observed in the characteristic polynomial $|\mathcal{L}(\xi) - \lambda \mathbb{1}|$, with $\alpha = (\beta_R - \beta_L)\epsilon + \beta_L \mu_L - \beta_R \mu_R$. For simplicity consider the case where $\Gamma_R = \Gamma_L$, $\beta_R = \beta_L$, $\mu_L = \epsilon + V/2$ and $\mu_R = \epsilon - V/2$.

Vorlesung:	• Do. 10:00 Uhr – 12:00 Uhr im EW 203. • Fr. 10:00 Uhr – 12:00 Uhr im EW 203.
Übung:	• Do. 16:00 Uhr –18:00 Uhr im EW 733.
Scheinkriterien:	• Mindestens 60% der Übungspunkte. • Regelmäßige und aktive Teilnahme am Tutorium.