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# 13. Übungsblatt – TPVI: Theorie des Quantentransportes

**Abgabe: Do. 13.02.2020 16:00 Uhr im Tutorium**

*Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen oder Dreiergruppen erfolgen.*

**Aufgabe 24 (30 Punkte):** *Feedback control of quantum transport*

Feedback and measurement permit to control the statistics of quantum transport. In particular, it is possible to freeze the growth of noise, thereby establishing a means for regular sources of intensity. For the case of unidirectional transport through a tunnel junction with no internal degrees of freedom, the master equation conditioned to the number $n$ of tunneled electrons has the form

$$\dot{\rho}^{(n)}(t) = \mathcal{L}^0_{(n)}(t)\rho^{(n)}(t) + \mathcal{J}_{(n-1)}(t)\rho^{(n-1)}(t), \tag{1}$$

where the jump and Liouville coefficients $\mathcal{L}^0_{(n)}(t) = -\mathcal{J}_{(n)}(t) = -\Gamma f(q_n(t))$ have been made dependent on the measurement outcome $n$ by means of the auxiliary function $f(q_n(t))$, with

$$q_n(t) \equiv I_0 t - n, \tag{2}$$

and $f(0) = 1$. This modulates the bare tunneling rate of electrons through the junctions $\Gamma$ multiplicatively proportionally to the deviation of the transported charge $n$ with respect to the average value without feedback $I_0 t$.

Let us consider the case of linear feedback, i.e. $f(x) = 1 + gx$, where $g > 0$ is the feedback strength.

(a) (6 Punkte) Calculate the average current with feedback

$$\langle n \rangle_t = \Gamma t, \tag{3}$$

and the variance

$$\langle n^2 \rangle_t - \langle n \rangle_t^2 = \frac{1}{2g}\left(1 - e^{-2g\Gamma t}\right), \tag{4}$$

directly from their definitions.

(b) (6 Punkte) By using the definition $\rho(\chi, t) \equiv \sum_n \rho^{(n)}(t) e^{i\chi n}$, transform the master equation (1) into the partial differential equation

$$\frac{\partial}{\partial t}\rho(\chi, t) = \mathcal{L}(\chi, t) f\left(I_0 t - \frac{\partial}{\partial i\chi}\right)\rho(\chi, t) \tag{5}$$

and find the form of $\mathcal{L}(\chi, t)$.

(c) (10 Punkte) Show that the cumulant generating function $C(\chi, t) \equiv \ln \rho(\chi, t)$ has the form

$$C(\chi, t) = I_0 t i\chi + \frac{1}{g}\ln\left[e^{i\chi}\left(1 - e^{-g\Gamma t}\right) + e^{-g\Gamma t}\right] + \frac{1}{g}\left[Li_2\left((1 - e^{-i\chi})e^{-g\Gamma t}\right) - Li_2\left(1 - e^{-i\chi}\right)\right],$$

where $Li_2(z) \equiv \int_z^0 \frac{dt}{t}\ln(1 - t)$.

(d) (8 Punkte) Show that in the long-time limit and for $k \geq 2$ the cumulants follow

$$\langle\langle n^k \rangle\rangle = -\frac{1}{g} B_{k-1} \qquad (6)$$

where the $B_k \equiv \frac{d^k}{dx^k} \frac{x}{e^x - 1}\Big|_{x=0}$ are the $k$th Bernoulli-Seki numbers.

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| Vorlesung: | • Do. 10:00 Uhr – 12:00 Uhr im EW 203.<br>• Fr. 10:00 Uhr – 12:00 Uhr im EW 203. |
| Übung: | • Do. 16:00 Uhr –18:00 Uhr im EW 733. |
| Scheinkriterien: | • Mindestens 60% der Übungspunkte.<br>• Regelmäßige und aktive Teilnahme am Tutorium. |